

LINEAR EQUATIONS WITH ONE VARIABLE

A common form of linear equation with one variable is :

$$a \cdot x = b$$

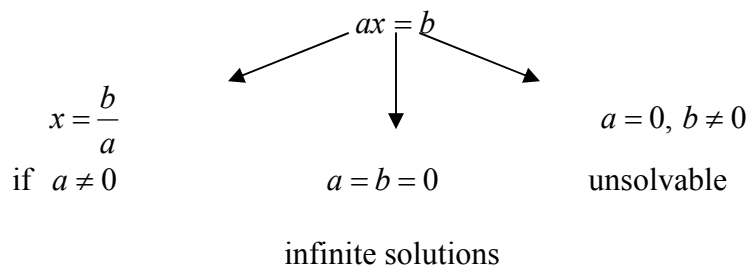
a and b are real numbers.

A solution x_0 is every real number for which is :

$$ax_0 = b$$

If we have: $A \cdot B = 0$ or $A \cdot B \cdot C = 0$ then:

$$\begin{aligned} A \cdot B = 0 &\Leftrightarrow A = 0 \vee B = 0 \\ A \cdot B \cdot C = 0 &\Leftrightarrow A = 0 \vee B = 0 \vee C = 0 \end{aligned}$$



Technique is to add, subtract, multiplu or divide both sides of the equation by the same number in order to isolate the variable on one side of the equation!

EXAMPLES

1) Solve the equation $9 - 2x = 5x + 2$

$$\boxed{9 - 2x = 5x + 2}$$

$$-2x - 5x = +2 - 9$$

$$-7x = -7$$

$$x = \frac{-7}{-7}$$

$$\boxed{x = 1}$$

2) Solve the equation: $3(2 - 3x) + 4(6x - 11) = 10 - x$

$$3(2 - 3x) + 4(6x - 11) = 10 - x$$

$$6 - 9x + 24x - 44 = 10 - x$$

$$-9x + 24x + x = 10 - 6 + 44$$

$$16x = 48$$

$$x = \frac{48}{16}$$

$$\boxed{x = 3}$$

3) Solve the equation : $\frac{y-5}{7} + 2 = \frac{2y-3}{2} - \frac{6y+5}{14}$

$$\frac{y-5}{7} + 2 = \frac{2y-3}{2} - \frac{6y+5}{14} \quad / \cdot 14$$

$$2(y-5) + 28 = 7(2y-3) - 1(6y+5)$$

$$2y - 10 + 28 = 14y - 21 - 6y - 5$$

$$2y - 14y + 6y = -21 - 5 + 10 - 28$$

$$-6y = -44$$

$$y = \frac{-44}{-6}$$

$$\boxed{y = +\frac{22}{3}}$$

4) Solve the equation : $(x+3)^2 - (x-4)^2 = 2x - 13$

$$(x+3)^2 - (x-4)^2 = 2x - 13$$

$$(x^2 + 6x + 9) - (x^2 - 8x + 16) = 2x - 13$$

$$\cancel{x^2} + 6x + 9 - \cancel{x^2} + 8x - 16 = 2x - 13$$

$$6x + 8x - 2x = -13 - 9 + 16$$

$$12x = -6$$

$$x = \frac{-6}{12}$$

$$\boxed{x = -\frac{1}{2}}$$

5) Solve the equation : $\frac{2}{x-2} = \frac{1}{x+3}$

pay attention: $x-2 \neq 0 \Rightarrow x \neq 2$ and $x+3 \neq 0 \Rightarrow x \neq -3$

$$\frac{2}{x-2} = \frac{1}{x+3}$$

$$2(x+3) = 1 \cdot (x-2)$$

$$2x + 6 = x - 2$$

$$2x - x = -2 - 6$$

$$\boxed{x = -8}$$

6) Solve the equation : $\frac{x+5}{3x-6} = \frac{1}{2} + \frac{2x-3}{2x-4}$

Conditions are: $3x-6 \neq 0$ and $2x-4 \neq 0$ that is $x \neq 2$

$$\frac{x+5}{3x-6} = \frac{1}{2} + \frac{2x-3}{2x-4}$$

$$\frac{x+5}{3(x-2)} = \frac{1}{2} + \frac{2x-3}{2(x-2)} \quad / \cdot 6(x-2)$$

$$2(x+5) = 3(x-2) + 3(2x-3)$$

$$2x+10 = 3x-6+6x-9$$

$$2x-3x-6x = -6-9-10$$

$$-7x = -25$$

$$\boxed{x = \frac{25}{7}}$$

7) Solve the equation : $\frac{2x-1}{2x+1} + \frac{8}{4x^2-1} = \frac{2x+1}{2x-1}$

$$\frac{2x-1}{2x+1} + \frac{8}{4x^2-1} = \frac{2x+1}{2x-1}$$

$$\frac{2x-1}{2x+1} + \frac{8}{(2x-1)(2x+1)} = \frac{2x+1}{2x-1} \quad / \cdot (2x-1)(2x+1)$$

$$(2x-1)^2 + 8 = (2x+1)^2$$

$$4x^2 - 4x + 1 + 8 = 4x^2 + 4x + 1$$

$$4x^2 - 4x - 4x^2 - 4x = 1 - 1 - 8$$

$$-8x = -8$$

$$x = 1$$

$$\begin{array}{ll} 2x+1 \neq 0 & 2x-1 \neq 0 \\ 2x \neq -1 & 2x \neq 1 \\ x \neq -\frac{1}{2} & x \neq \frac{1}{2} \end{array}$$

8) Solve the equation : $|5x-1| + x = 2$

$$|\otimes| = \begin{cases} \otimes, \otimes \geq 0 \\ -\otimes, \otimes < 0 \end{cases} \quad |5x-1| = \begin{cases} 5x-1, \text{ if } 5x-1 \geq 0 \\ -(5x-1), \text{ if } 5x-1 < 0 \end{cases} = \begin{cases} 5x-1, & x \geq \frac{1}{5} \\ -(5x-1), & x < \frac{1}{5} \end{cases}$$

We have two equations:

$$\underline{\text{if } x \geq \frac{1}{5}}$$

$$5x-1+x=2$$

$$6x=2+1$$

$$6x=3$$

$$x = \frac{3}{6}$$

$$x = \frac{1}{2}$$

$$\underline{\text{if } x < \frac{1}{5}}$$

$$-(5x-1)+x=2$$

$$-5x+1+x=2$$

$$-4x=2-1$$

$$-4x=1$$

$$x = -\frac{1}{4}$$

9) Solve the equation : $|x-4|-|2x+3|=2$

$$|x-4| = \begin{cases} x-4, & x-4 \geq 0 \\ -(x-4), & x-4 < 0 \end{cases} = \begin{cases} x-4, & x \geq 4 \rightarrow I \\ -(x-4), & x < 4 \rightarrow II \end{cases}$$

$$|2x+3| = \begin{cases} 2x+3, & 2x+3 \geq 0 \\ -(2x+3), & 2x+3 < 0 \end{cases} = \begin{cases} 2x+3, & x \geq -\frac{3}{2} \rightarrow III \\ -(2x+3), & x < -\frac{3}{2} \rightarrow IV \end{cases}$$

We will divide task in 4 parts:

I and III

$$x \geq 4 \text{ i } x \geq -\frac{3}{2}$$

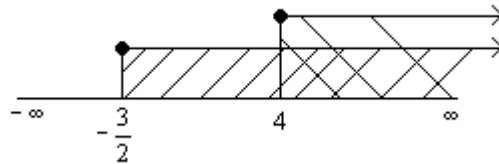
$$(x-4)-(2x+3)=2$$

$$x-4-2x+3=2$$

$$-x=2+4+3$$

$$-x=9$$

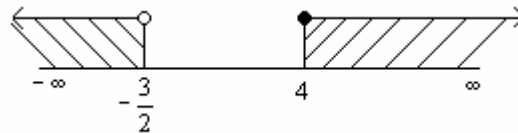
$$x=-9 \text{ is not a solution because } x \in [4, \infty)$$



$$x \in [4, \infty)$$

I and IV

$$x \geq 4, x < -\frac{3}{2}$$



unsolvable

$$x \in \emptyset$$

II and III

$$x < 4 \text{ i } x \geq -\frac{3}{2}$$

$$-(x-4)-(2x+3)=2$$

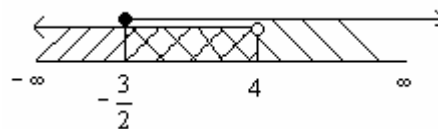
$$-x+4-2x-3=2$$

$$-3x=2+3-4$$

$$-3x=1$$

$$x = -\frac{1}{3}$$

$$-\frac{1}{3} \in \left[-\frac{3}{2}, 4\right)$$



$$x \in \left[-\frac{3}{2}, 4\right)$$

II and IV

$$x < 4, \text{ i } x < -\frac{3}{2}$$

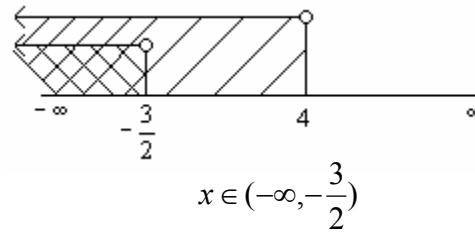
$$-(x-4) + (2x+3) = 2$$

$$-x+4+2x+3=2$$

$$x=2-4-3$$

$$x=-5$$

$$-5 \in \left(-\infty, -\frac{3}{2}\right)$$



Solutions are $x_1 = -\frac{1}{3}$ and $x_2 = -5$

9) Solve the equation : $mx - 3m = 1 + 5x$ $m \in R$

$$mx - 3m = 1 + 5x$$

$$mx - 5x = 1 + 3m$$

$$x(m - 5) = 1 + 3m$$

$$x = \frac{1 + 3m}{m - 5}$$

If $m = 5 \Rightarrow x = \frac{1 + 3 \cdot 5}{0} \Rightarrow$ **unsolvable**

If $m \neq 5 \Rightarrow x = \frac{1 + 3m}{m - 5} \Rightarrow$ **undetermined, because $m \in R$, ∞ solutions**

10) Solve the equation : $2ax - a + 4 = 8a + 7 - 5x$

$$2ax - a + 4 = 8a + 7 - 5x$$

$$2ax + 5x = 8a + 7 + a - 4$$

$$x(2a + 5) = 9a + 3$$

$$x = \frac{9a + 3}{2a + 5}$$

If $2a + 5 = 0 \Rightarrow a = -\frac{5}{2}$ **unsolvable**

If $2a + 5 \neq 0 \Rightarrow$ **undetermined, ∞ solutions**

Equations have big application in “problem” tasks. It is important to study the text, if there is a need, make a setting of problem and find a connection between data.

11) Father has 43 year and son 18. For how many years father will be two times older than son?

Mark with x - number of years that have to pass.

Father – 43 years

Son- 18 years

How years are passing for father and for son...we have:

Father – $43+x$ years

Son - $18+x$ years

$$2 \cdot (18 + x) = 43 + x$$

$$36 + 2x = 43 + x$$

$$2x - x = 43 - 36$$

$$\boxed{x = 7}$$

12) Student read one day $\frac{1}{4}$ of the book, second day $\frac{2}{3}$ from the rest of the book, third day last 40 pages.

How many pages has that book?

Mark with x - number of pages.

$$\frac{1}{4}x \rightarrow \text{first day}$$

$$\frac{2}{3} \cdot \frac{3}{4}x \rightarrow \text{second day}$$

$$40 \text{ str.} \rightarrow \text{third day}$$

$$\frac{1}{4}x + \frac{2}{\cancel{3}} \cdot \frac{\cancel{3}}{4}x + 40 = x$$

$$\frac{1}{4}x + \frac{2}{4}x + 40 = x$$

$$\frac{3}{4}x + 40 = x$$

$$x - \frac{3}{4}x = 40$$

$$\frac{1}{4}x = 40$$

$$x = 160$$

13) One worker can finish a job in 9, and second in 12 days. If there is a third worker , they will finish the job in 4 days. For how long will the third worker finish the job alone?

x- is time in wich third worker finish a job.

How do we think?

If the first finish a job in 9 days, then for 1 day he will do $\frac{1}{9}$ of job.

Similar , second worker will do $\frac{1}{12}$ of job and third $\frac{1}{x}$ of job.

It means that they togeder for one day do $\frac{1}{9} + \frac{1}{12} + \frac{1}{x}$ part of job. They are working 4 days, so:

$$\left(\frac{1}{9} + \frac{1}{12} + \frac{1}{x}\right) \cdot 4 = 1$$

$$\frac{4}{9} + \frac{4}{12} + \frac{4}{x} = 1 \quad \dots\dots\dots / \cdot 36x$$

$$16x + 12x + 144 = 36x$$

$$28x - 36x = -144$$

$$-8x = -144$$

$$\boxed{x = 18}$$

The third worker will finish job for 18 days!